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A SIMPLE QUASI-THREE DIMENSIONAL MODEL OF LONGSHORE CURRENTS OVER ARBITRARY PROFILE

by

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The longshore current maximum observed in the trough of a barred beach during the nearshore dynamics experiment DELILAH at Duck, North Carolina, is not predicted by present theory. The simplest longshore current models balance cross-shore changes in the alongshore wave momentum (radiation stress) with the alongshore bottom shear stress. Waves break over the bar, reform in the trough and again break on the foreshore resulting in changes in the radiation stress, which predicts two jets, one over the bar and the other at the foreshore, which does not agree with the observed current maximum in the trough. The advection of the momentum of the longshore current by mean cross-shore currents as a source of momentum mixing is investigated. The longshore current is strongest toward the surface and decreasing to zero at the bottom. The cross-shore mean current has an onshore transport in the wave crest/trough region and an offshore transport beneath (undertow). The net interaction can induce significant lateral mixing of the alongshore momentum of the mean currents, which is shown using a simplified three-dimension model of nearshore currents to explain much of the differences with observations.

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A SIMPLE QUASI-THREE DIMENSIONAL MODEL OF LONGSHORE CURRENTS OVER ARBITRARY PROFILE

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Submitted in partial fulfillment
of the requirements for the degree of

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from the

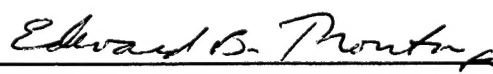## NAVAL POSTGRADUATE SCHOOL
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ABSTRACT

The longshore current maximum observed in the trough of a barred beach during the nearshore dynamics experiment DELILAH at Duck, North Carolina, is not predicted by present theory. The simplest longshore current models balance cross-shore changes in the alongshore wave momentum (radiation stress) with the alongshore bottom shear stress. Waves break over the bar, reform in the trough and again break on the foreshore resulting in changes in the radiation stress, which predicts two jets, one over the bar and the other at the foreshore, which does not agree with the observed current maximum in the trough. The advection of the momentum of the longshore current by mean cross-shore currents as a source of momentum mixing is investigated. The longshore current is strongest toward the surface and decreasing to zero at the bottom. The cross-shore mean current has an onshore transport in the wave crest/trough region and an offshore transport beneath (undertow). The net interaction can induce significant lateral mixing of the alongshore momentum of the mean currents, which is shown using a simplified three-dimension model of nearshore currents to explain much of the differences with observations.

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I. INTRODUCTION

The longshore current maximum observed in the trough of the barred beach during the nearshore dynamics experiment DELILAH at Duck, North Carolina, is not predicted by present theory. The simplest longshore current models balance cross-shore changes in the alongshore wave momentum (radiation stress) with the alongshore bottom shear stress. Waves break over the bar, reform in the trough and again break on the foreshore. Wave breaking results in changes in the radiation stress predicting two jets, one over the bar and the other at the foreshore, which does not agree with the observed current maximum in the trough. An example of the measured and modeled wave height and longshore current distributions are shown in Figure 1. The predictions suggest that a transfer of momentum is required to account for the current deficit in the trough.

A number of mechanisms have been proposed to mix momentum laterally into the trough region to drive the longshore current. Traditional turbulent mixing, usually parameterized using classical eddy viscosity concepts associated with the shear of the longshore current (e.g., Bowen, 1969; Longuett-Higgins, 1970; Thornton, 1970), would require up gradient mixing of the longshore current which is not feasible. Battjes (1975) formulated turbulent mixing induced by breaking waves. The scale of the turbulent mixing was the same order as the wave height, which is much too short a length scale to explain the observations. Smith et al (1993) described wave breaking as rollers that propagate with the wave at the phase speed; they applied a turbulent kinetic energy equation and argued that turbulence was diffused downward into the water column generating an additional alongshore thrust. They applied their formulation to the

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Wave Height (H_rms) Transformation

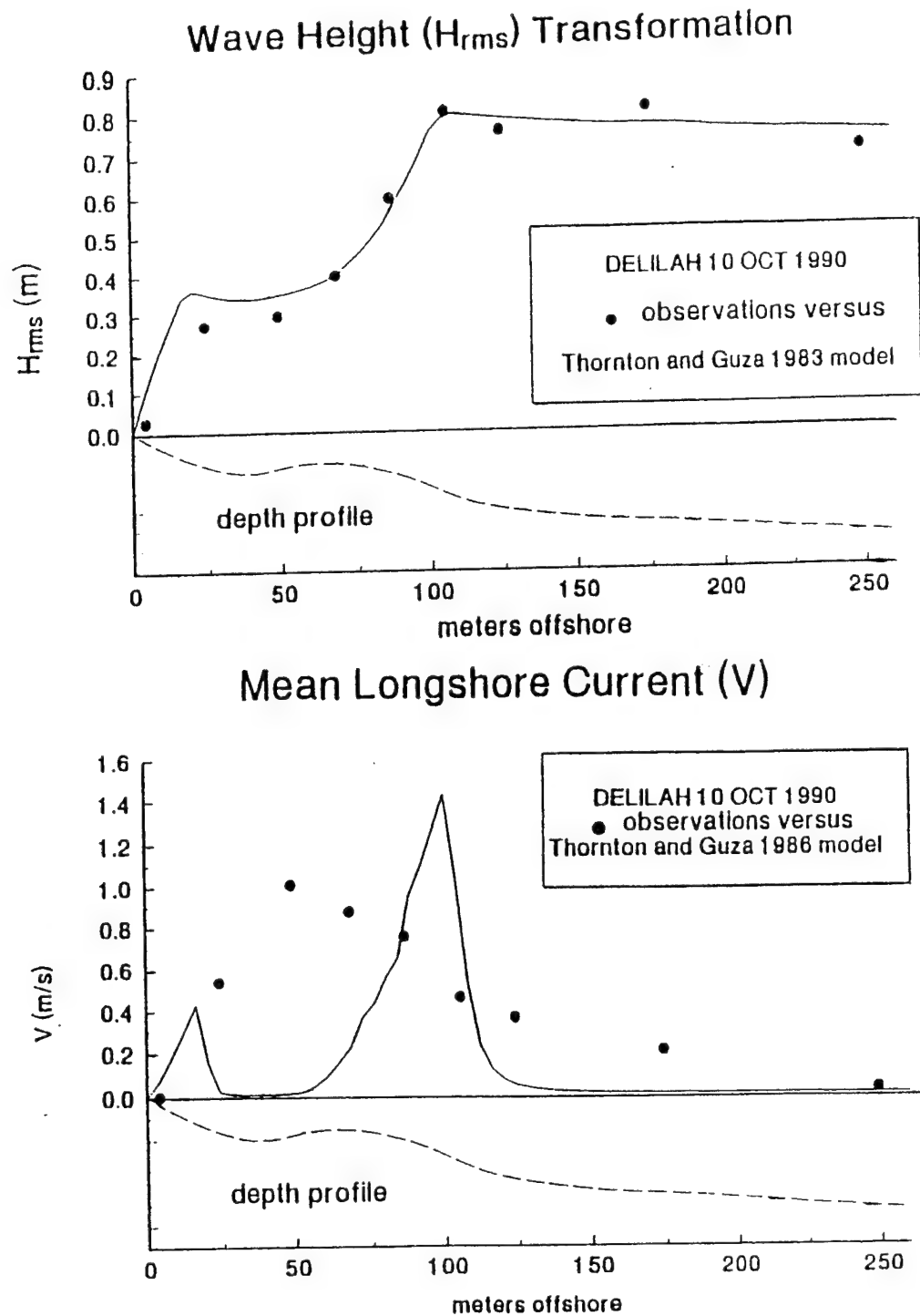## Mean Longshore Current (V)

Figure 1. Cross-shore Distributions of H_{rms} Wave Heights (Upper Panel) and Mean Longshore Currents (Lower Panel). The Solid Circles are the Measured Values on 10 October 1990 During DELILAH Experiment. The Solid Lines are the Thornton and Guza (1986) Model Predictions.

DELILAH data and partially explained the momentum deficit in the trough.

Changes in the bottom shear stress due to turbulence being injected from the surface by breaking waves and modifying the vertical distribution of the longshore current, which in turn modify the magnitude of the longshore current, were investigated by Church and Thornton (1993). They allow a spatially variable bed shear stress coefficient dependent on the breaking wave induced near bottom turbulence levels. The model predicted cross-shore profiles of the longshore current improved agreement with observation compared with treatments using constant bed shear stress values, but did not completely account for the momentum deficit in the trough.

Instabilities of the longshore current have been identified as a mechanism for turbulent mixing of the longshore current originally suggested by Bowen and Holman, (1989). Dodd and Thornton (1990) showed that if shear instabilities exist, there is an accompanying cross-shore mixing of momentum. Putrevu and Svendsen (1992) carried out a numerical study of shear instabilities over various topography, and using an order of magnitude analysis concluded that even a weak shear in the longshore current might be capable of producing significant mixing. Significant shear instabilities of the longshore current have been observed in the field (Oltman-Shay et al., 1989). Church et al. (1995) calibrated the amplitude of the shear instabilities using field measurements, and then calculated the Reynolds' stress associated with the instabilities. They found that the mixing predicted due to shear instabilities to be in qualitative agreement with that required for modeled longshore current profiles to agree with observed profiles.

The advection of the momentum of the longshore current by mean cross-shore currents as a source of momentum mixing was suggested by Putrevu and Svendsen (1993). The longshore current is strongest toward the surface and decreases to zero at the bottom. The cross-shore mean

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current has an onshore transport in the wave crest/trough region and an offshore transport below (undertow). Putrevu and Svendsen (1994) showed that the net interaction could induce significant mixing of the momentum of the mean currents.

The objective of this paper is to formulate a simplified three-dimension model to describe longshore currents, including turbulent mixing due to the cross-shore advection of mean momentum of the alongshore current by the shear of the mean cross-shore current as suggested by Svendsen and Putrevu (1994). The observations acquired during DELILAH and DUCK94 will be used to test the model predictions.

II. THEORY

A three-dimensional model of the nearshore circulation is derived assuming stationary wave conditions, straight and parallel bottom contours, and Gaussian distributed random waves which are narrow-banded in both frequency and direction (i.e., all waves are from the same direction and of a single frequency). A right-handed-coordinate system is used with x positive offshore, y alongshore and z positive upward from the sea surface. In the following, the random wave distributions are described, which is used to ensemble average the various equations. The conservation of mass flux is described next, which provides an integral condition for the solution of the vertical profile of the cross-shore velocity. The depth integrated cross-shore and alongshore momentum equations are considered next. The cross-shore momentum equation describes wave set-up/down, which is the primary forcing of the cross-shore undertow. The alongshore momentum equation contains the lateral transfer of mean momentum term, which requires specifying the vertical profile of U(x,z) by solving the vertical momentum equation. The alongshore momentum equation is then solved to find the longshore current profile, V(x).

A.   RANDOM WAVE DISTRIBUTION

The random wave heights are assumed Rayleigh distributed, p(H), everywhere even when breaking, which was shown to be a reasonable assumption at least for mild sloping beaches by Thornton and Guza (1983). The ensemble averaged wave energy is calculated by integrating E_w given by linear theory across the Rayleigh distribution to give:

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$$<E_w> = b\rho g H_{rms} \qquad \qquad \textbf{(1)}$$

where $<\;>$ indicates ensemble averaging.

B. CONSERVATION OF MASS FLUX

The conservation of mass flux for straight and parallel contours is given by

$$\overline{\int_{-h}^{\eta} \rho[U(z) + \tilde{u}(z)] \, dz} = 0 \qquad \qquad \textbf{(2)}$$

where the boundary condition of no flow through the beach at $x = 0$ has been utilized and the velocities have been partitioned into mean and wave contributions. Equation 2 states that onshore mass transport by waves is balanced locally by an offshore transport. In an Eulerian reference frame, there is an onshore mass transport by waves limited to an upper region between the crest and trough. Describing waves using linear theory, and expanding in a Taylor series about the mean water level, $\overline{\eta}$ (Phillips, 1977), the onshore mass transport in the upper region is given by

$$<q_x> = \overline{\int_{\eta_t}^{\eta_c} \rho \tilde{u}(z) \, dz} = \frac{\rho g}{8C}<H^2> \qquad \qquad \textbf{(3)}$$

where subscripts (c, t) refer to (crest, trough), and velocities are uniform over the crest-trough

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region to second order. The term on the rhs is the ensemble averaged wave contribution to the mass flux. The onshore mass transport in the upper region is balanced by a return flow below the trough (undertow)

$$<q_x> = - \int_{-h}^{\eta_t} \rho U(z) dz = -\rho U_r h_t \tag{4}$$

where an equivalent mean uniform return flow (U_r) is defined and h_t is the depth below the trough.

C.   DEPTH INTEGRATED MOMENTUM FLUX

The depth integrated cross-shore momentum equation is given by

$$\frac{\partial \tilde{S}_{xx}}{\partial x} = -\rho g (\overline{\eta} + h) \frac{\partial \overline{\eta}}{\partial x} \tag{5}$$

where $\tilde{S}_{xx}$ is the wave cross-shore radiation stress and $\overline{\eta}$ is the mean water level (set-up/down.)

The depth averaged alongshore momentum flux equation is given by

$$\frac{\partial \tilde{S}_{yx}}{\partial x} + \frac{\partial}{\partial x}[\int_{-h}^{\eta_t} \rho VU \, dz + \overline{\int_{\eta_t}^{\eta_c} \rho(\tilde{u} \, V + \tilde{v} \, U) dz}] = -\tau_y^b \tag{6}$$

where the surface wind stress is neglected, and the alongshore bottom stress is given by

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$$\tau_y^b = \rho c_f \overline{|\vec{u}|(V + \vec{v})} \tag{7}$$

where c_f is a bed shear stress coefficient.

D.   VERTICAL PROFILE OF LONGSHORE VELOCITIES

Assuming constant shear stress and Prantl mixing length theory, the vertical profile of the longshore current is given by

$$V(z) = \frac{v_*}{\kappa} \ln(\frac{z + h}{z_a}) \tag{8}$$

where the von Karman constant, $\kappa = 0.4$, and z_a is an apparent roughness due to the increase in effective roughness owing to the presence of waves (e.g., Grant and Madsen, 1979) such that $V(z = -h + z_a) = 0$. The vertical profile of the longshore current is parameterized by v_* and z_a, where the shear velocity is defined by

$$\tau_y^b = \rho v_*^2 \tag{9}$$

To compare the predicted longshore currents with the measured values, the mean longshore current over the vertical is defined as the reference velocity given by

$$V_m(x) = \frac{1}{h} \int_{-h+z_a}^{0} \frac{v_*}{\kappa} \ln(\frac{z+h}{z_a}) dz = \frac{v_*}{\kappa} [\ln\frac{h}{z_a} + \frac{z_a}{h} - 1] \qquad (10)$$

Since bottom roughness was not measured during the DELILAH experiment, it is more convenient to rewrite V(z) by relating v_* to V_m and z_a to a bed shear stress coefficient c_f, whose value for model applications has been previously calculated (Thornton and Guza, 1986; Church and Thornton, 1993; Whitford and Thornton, 1995; Thornton et al, 1995.) Using Equations 7 and 9

$$\tau_y^b = \rho v_*^2 = \rho c_f \overline{|\vec{u}|} V_m \qquad (11)$$

giving

$$V(z) = V_m + (\frac{c_f |u| V_m}{\kappa^2})^{\frac{1}{2}} [\ln(1 + \frac{z}{h}) + 1] \qquad (12)$$

where it has been assumed $z_a \ll h$.

E.   VERTICAL PROFILE OF CROSS-SHORE VELOCITIES

The monochromatic wave formulation for undertow by Stive and Wind (1986) is extended using the random wave formulation of Thornton and Guza (1983). The vertical momentum equation is given by

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$$-\frac{\partial}{\partial z}\rho(\overline{u'w'}) = \frac{\partial}{\partial z}\rho v_t\frac{\partial}{\partial z}U(z) = \frac{\partial R}{\partial x} \qquad (13)$$

where

$$R = \rho[g\overline{\eta} + \frac{1}{2}(\overline{\tilde{u}^2} - \overline{\tilde{v}^2})] \qquad (14)$$

and can be shown to be independent of depth using linear wave theory. The forcing of the undertow is due primarily to the gradient of the wave set-up and the radiation stress gradient terms in R. Integrating twice and solving for the integration constants by applying conservation of the mass (Equation 4) and equating stress across the trough level (by integrating the vertical momentum equation from the bottom to trough elevation) gives

$$U(z) = U_r(H) + (-C_0 + C_1 z + C_2 z^2)\frac{\partial R}{\partial x} + \tau_x^b[1 + \frac{1}{2\rho\eta_t}(\eta_t - h)] \qquad (15)$$

where

$$C_0 = \frac{1}{6\rho v_t}(h_t^2 - 3h^2) \qquad (16)$$

$$C_1 = \frac{h}{\rho v_t} \qquad (17)$$

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$$C_2 = \frac{1}{2\rho\nu_t} \tag{18}$$

The trough depth is approximated as half the rms wave height, i.e., $\eta_t \approx H_{rms}/2$, and $h_t = \overline{\eta_t} + h$. The bottom stress terms in Equation 15 are negligibly small inside the surf zone and are neglected. Thus, ensemble averaging Equation 15 gives

$$<U(z)> = <U_r(H)> + (- C_0 + C_1 z + C_2 z^2) <\frac{\partial R}{\partial x}> \tag{19}$$

The forcing of the undertow within the surf zone is due to the pressure gradient of the set-up and the onshore radiation stress gradient given by

$$<\frac{\partial R}{\partial x}> = \rho g b [\frac{1}{h}\frac{\partial <H^2>}{\partial x} + \frac{<H^2>}{2}\frac{\partial(h^{-1})}{\partial x}] \tag{20}$$

where $<H^2> = H_{rms}^2$ from the Rayleigh distribution.

The set-up/down in R (Equation 14) is solved by applying a centered finite differencing scheme to Equation 5, and assuming that the wave set-up/down is zero at the most off-shore grid point. After determining the set-up/down, its cross-shore gradient can also be determined by using a centered finite differencing scheme.

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F.   MOMENTUM TRANSFER BY MEAN CURRENTS

Mixing is the result of the shear between the cross-shore and longshore mean currents as described by the 2nd and 3rd terms in Equation 6. In the surface region between the crest and trough of the wave, the alongshore momentum of the mean longshore current, ρV, is advected shoreward due to the mean mass transport velocity of the waves. In the water column beneath the trough, ρV is advected offshore by the undertow. Beneath the trough

$$\int_{-h}^{\eta_t} \rho VU dz = \rho <U_r> \frac{v_*}{\kappa} h_t [\ln(\frac{h_t}{z_a}) - 1] + \frac{v_*}{6\kappa v_t}(\frac{2}{3}h_t^{3}) <\frac{\partial R}{\partial x}> \tag{21}$$

Combining Equations 21 and 10

$$\int_{-h}^{\eta_t} \rho VU dz = \rho <U_r> h_t V_m + \frac{v_*}{9\kappa v_t} h_t^{3} <\frac{\partial R}{\partial x}> \tag{22}$$

A transfer of momentum in the crest/trough region is due to the interaction of the mass transport velocity by waves with the longshore current, third term in Equation 6, and can be simplified because the angle of wave incidence is generally small and the radiation stress contributions ($\rho \tilde{u}\tilde{v}$) in the crest-trough region are of higher order. Thus

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$$\overline{\int_{\eta_t}^{\eta_c} \rho(\tilde{u}V + \tilde{v}U)\,dz} \approx \overline{\rho V(\overline{\eta} \approx 0)\tilde{u}(\overline{\eta} \approx 0) \int_{\eta_t}^{\eta_c} dz} = <q_x> \frac{v_*}{\kappa} \ln \frac{h}{z_a} \tag{23}$$

G.   LONGSHORE CURRENTS

The cross-shore distribution of mean longshore currents is solved using Equation 6. The longshore currents were measured near mid-depth, but at arbitrary elevations, z_m. To make comparisons with the model, the measured longshore currents, V_{meas}, are corrected to correspond to the reference mean longshore current V_m using

$$V_{mm} = \frac{V_m}{V(z=z_m)} V_{meas}(z=z_m) = \frac{(\ln \frac{h}{z_a} + \frac{z_a}{h} - 1)}{\ln(h - \frac{z_m}{z_a})} V_{meas}(z=z_m) \tag{24}$$

.

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III. MODEL DEVELOPMENT

The objective is to obtain a numerically stable scheme that can be applied to predict the mean longshore current, V_m, defined as our reference velocity (Equation 10), using the depth averaged alongshore momentum flux, Equation 6.

A.   MODEL FORMULATION

The first step is to rewrite Equation 6 in terms of V_m. The first term on the lhs, cross-shore gradient of alongshore radiation stress, is the forcing term; thus, it is a not a function of our reference velocity. Using linear wave theory and shallow water assumption, we obtain

$$F(x) = \frac{b \rho g^{3/2} \sin\alpha_o}{c_0} \frac{\partial}{\partial x} [H_{rms}^2 h^{1/2} \cos\alpha] \tag{25}$$

The remaining terms of this equation are all functions of the reference velocity. The second term on the lhs is the momentum flux gradient due to the mean currents. It can be divided in two terms, where the first term is the gradient from the bottom up to the trough level, with the second term in the region between the trough and the crest. The term on the rhs is the bottom stress. Again applying linear wave theory, and the predicted vertical profiles of mean cross-shore current (Equation 15) and mean longshore current (Equation 12) as a function of the reference velocity, these terms can be rewritten as

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$$\frac{\partial}{\partial x} \int_{-h}^{\eta_t} \rho VU \, dz = K_1(x) V_m^{1/2} \tag{26}$$

$$\frac{\partial}{\partial x} \overline{\int_{\eta_t}^{\eta_c} \rho(\tilde{u} \, V + \tilde{v} \, U) dz} = \frac{K_2(x)}{V_m^{1/2}} \frac{\partial V_m}{\partial x} \tag{27}$$

$$-\tau_y^b = K_3(x) V_m \tag{28}$$

where the above coefficients K $_1$(x), K $_2$(x), and K $_3$(x) are given by

$$K_1(x) = 2K_2(x) \left[\frac{1}{A(x)} \frac{\partial A(x)}{\partial x} + \frac{1}{2H_{rms}(x)} \frac{\partial H_{rms}(x)}{\partial x} - \frac{1}{4h(x)} \frac{\partial h(x)}{\partial x} \right] \tag{29}$$

$$K_2(x) = \frac{1}{2} \sqrt{\frac{C_f}{2}} \sqrt{\frac{g}{\pi}} \frac{H_{rms}^{1/2}(x) A(x)}{h^{1/4}(x)} \tag{30}$$

$$K_3(x) = -\frac{\rho C_f}{2} \sqrt{\frac{g}{\pi}} \frac{H_{rms}(x)}{h^{1/2}(x)} \tag{31}$$

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and

$$A(x) = [\frac{1}{9 \kappa v_t} h_t^{3}(x) < \frac{\partial R(x)}{\partial x}> + \frac{<q_x(x)>}{\kappa}]$$ (32)

Substituting Equations 25 thru 28 into Equation 6 and rearranging the terms, we obtain a first order nonlinear partial differential equation for the reference velocity V_m

$$\frac{\partial V_m}{\partial x} = \frac{V_m^{1/2}}{K_2}[K_3 V_m - K_1 V_m^{1/2} - F]$$ (33)

B.   NUMERICAL SCHEME

The Newton-Raphson method, applying a centered finite differencing scheme to rewrite the partial derivative, is used to calculate V_m . This method is a vectorized form of the Newton iterative method for solving nonlinear partial differential equations. An analogy can be made between this method and the "shooting method," as the behavior of the latter is more easily visualized. A simplified description of the necessary steps to apply the "shooting method" for determining the cross-shore variability of V_m is:

1. First guess for V_m at the most offshore point of the grid;

2. Applying Equation 26 to determine the cross-shore distribution of V_m;

3. Compare the model output at the shoreline with the physically determined boundary condition, no longshore velocity at the beach ($V_m = 0$ at $x = 0$);

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4. If the difference exceeds a predetermined tolerance, change the first guess, and restart the process;

5. Repeat steps 1 to 4 until the tolerance is reached.

In spite of its simplicity, the "shooting method" is a slowly converging scheme, and does not allow the inclusion of a second boundary condition (e.g., no longshore velocity at the most offshore grid point.) Some of the advantages of applying the Newton-Raphson method, is that it is a stable, and fast convergent method. Another important characteristic of this method is its vectorized form, which allows the comparison of the new calculated cross-shore distribution of V_m with the previous one at all the grid points across the surf zone, not just at the shoreline.

C.   BOUNDARY CONDITIONS

The two natural choices for the boundaries are the shoreline ($x = 0$) and the most offshore point where data were collected ($x = \infty$). At these boundaries, it is assumed that there was no wave forcing so that the longshore velocity vanishes.

D.   INITIAL CONDITIONS

Due to its vectorized form, the Newton-Raphson method requires an initial vector for the cross-shore distribution of the reference velocity ($V_m (x)$). The natural choice would be a motionless state, but as it generates numerical problems (division by zero), the initial condition used was a small, constant, non-zero value for $V_m (x.)$ The sensibility of the model to the first

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guess field is limited to the number of iterations it takes to reach the steady state due to the applied forcing.

E.   MODEL INPUTS

The required inputs can be divided into three categories. The first is the observed data of incident wave angle (α_O) and peak frequency (f_p), and bottom profile (h). The second is input from the Thornton and Guza (1983) model for the cross-shore variation of H_{rms}. The final are the parameters that should be constrained by data, which include the vertical eddy viscosity (v_t) defining the undertow and bottom stress and the bottom stress coefficient (C_f) defining the longshore current profile. Both parameters are assumed constant across the surf zone.

IV. FIELD EXPERIMENTS AT DUCK

The data were acquired as part of the DELILAH (October 1990) and DUCK94 (October 1994) experiments conducted at the U.S. Army Corps of Engineers Field Research Facility (FRF) at Duck, North Carolina. Offshore directional wave spectra were measured in both experiments using a linear array of 10 pressure sensors in 8m depth. During DELILAH the cross-shore transect of wave heights and currents were measured using a cross-shore array of two-component Marsh-McBirney (model 512) electromagnetic current meters and pressure sensors at each of 9 locations. The data were sampled at 8 Hz. The location and elevation of the current meters relative to selected beach profiles are shown in Figure 2. Mean tidal range in the study area is less than 1m. The cross-shore array was designed to measure more intensely over the bar where the largest changes in the wave height occur due to wave breaking. The current meters were kept at the same elevation relative to mean-sea-level, with the exception of current meter 1 whose elevation had to be adjusted during the experiment to accommodate changes in the beach profile. The top of the bar was near sensor 3 early in the experiment and moved offshore to sensor 5 in response to the increased wave height commencing on 10 October 1990. During DUCK94, wave heights and currents were measured using a moveable sled. The sled initially was towed offshore, and then towed onshore stopping at offshore the bar, on the bar and in the trough to make measurements for at least one hour at five to eight locations each day. The sled was instrumented with a wave directional array of six pressure sensors and eight electromagnetic current meters located at elevations 0.23, 0.41, 0.68, 1.01, 1.46, 1.79, 2.24 and 2.57m above the bed.
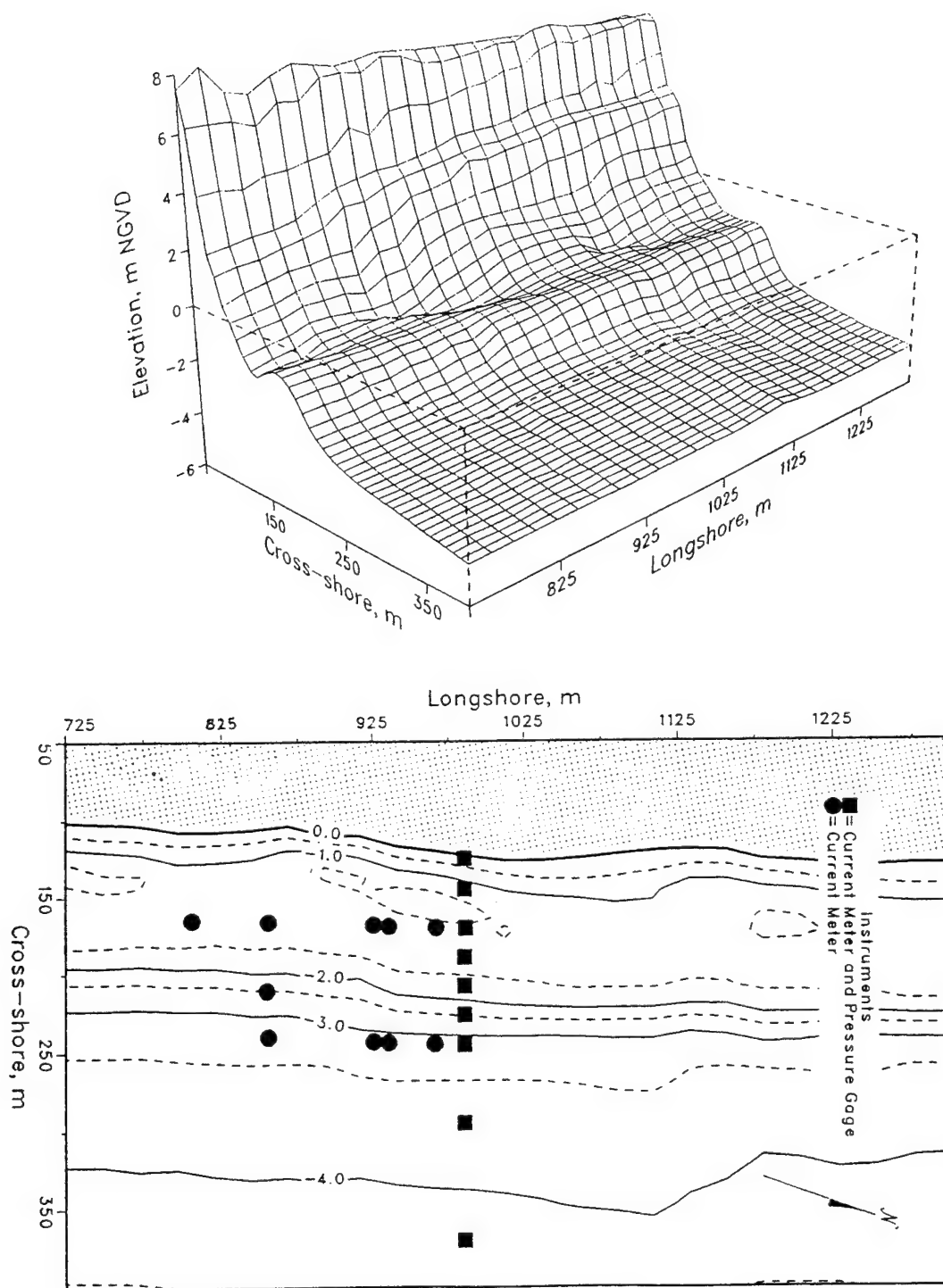
Figure 2. Bottom Profile During DELILAH Experiment on 10 October 1990 with Current Meter

Locations Indicated.

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A wide variety of wave conditions occurred during these experiments including northeasters, swell from the south, a frontal system from the south, and hurricane waves up to 2.5m. The resulting longshore currents were generally strong, ranging up to 1.5 m/sec. The examples presented here from DELILAH are during times in which moderate swell waves were incident at relatively large angles driving a strong longshore current.

The bar at this location is highly mobile and responded to the variable wave conditions during the experiment. The changing of the bar location has a significant effect on the resulting wave height and longshore current profiles. The bar tends to be three-dimensional, or rhythmic, during times of moderate waves and becomes linear during times of storms and associated strong longshore currents. For the cases presented here, a well developed, nearly linear bar was present. H_{rms} values were calculated using the variance, σ^2, of the surface elevation spectra across the wind-wave band of frequencies (0.05 to 0.3 Hz), and assuming the waves are Raleigh distributed, such that $H_{rms} = (8\sigma^2)^{1/2}$. The surface elevation spectra were calculated by converting pressure data using a linear theory transfer function.

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V. DISCUSSION AND CONCLUSIONS

Vertical distributions of currents are examined using the DUCK94 data. The data from 11 October 1994 was selected during a time of high onshore winds associated with short period (T_p = 7 sec) incident waves from the north at a mean angle of about 16 degrees just offshore the bar. The cross-shore mean currents are predicted using Equation 15. The wave heights are first predicted using the wave transformation model by Thornton and Guza (1986), which then are used to calculate the radiation stress gradients. The radiation stress gradients are the forcing terms required to solve for the wave set-up/down gradients (Equation 5) in order to describe the forcing term for the cross-shore currents and the longhore currents (Equation 6). The predicted wave set-up/down is shown in the upper panel of Figure 3.

The cross-shore flow is onshore in the crest-tough region and offshore beneath (undertow). The predicted equivalent mean uniform return flow at various cross-shore locations are shown in Figure 4 and are compared with measurements. The model appears to under predict the strength of the onshore transport and consequently the return flow that is determined locally by applying mass conservation over the vertical. The mean uniform return flow and measurements are in at least qualitative agreement, except for the region over the bar where the strong undertow jet is under estimated by this model.

The vertical profiles of the mean longshore current predicted using Equation 8 are shown in Figure 5. In the lower water column the longshore current is represented by the log profile and in the crest-trough region the log profile is modified since the averaged measured velocity is decreased as the current meter is out of the water part of the time. To account for this, the

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SET-UP (m)

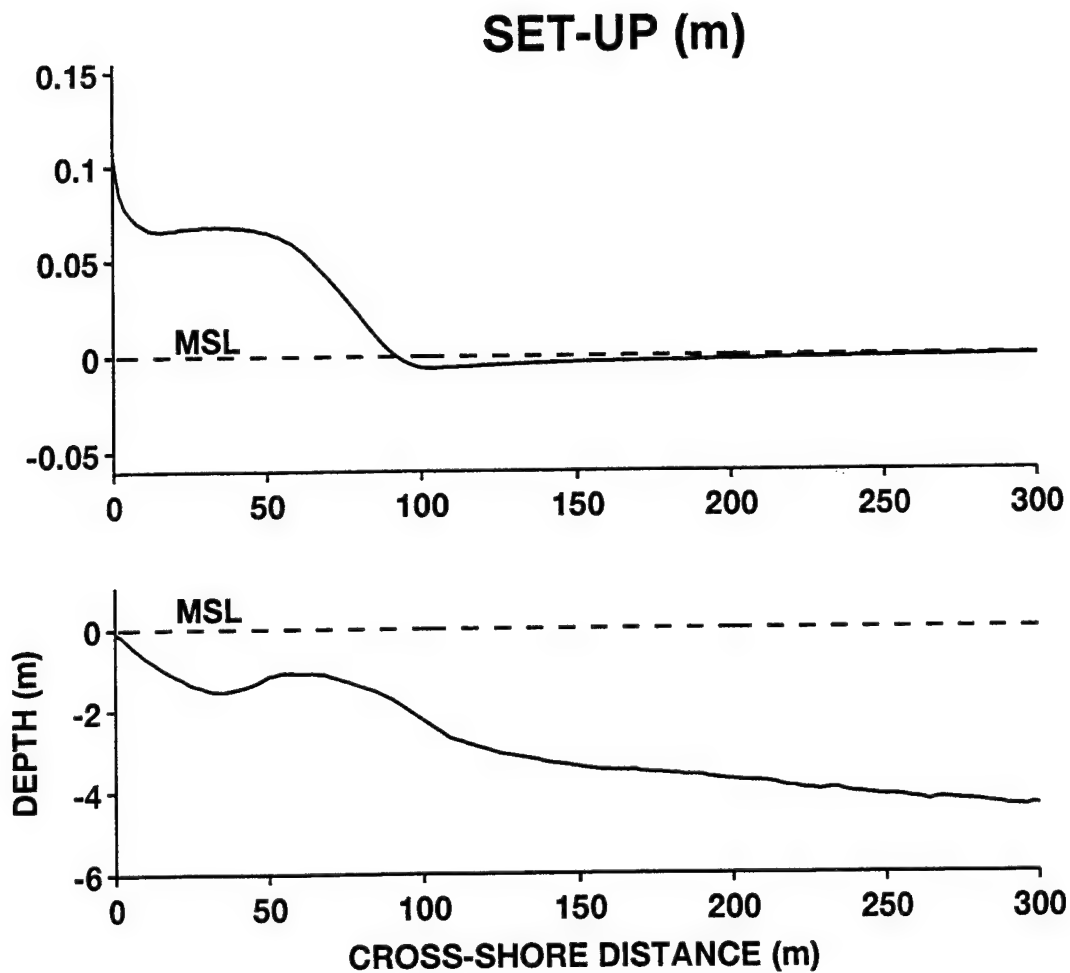Figure 3. Predicted Wave Set-up/down (Upper Panel), and Respective Bathymetry (Bottom Panel) During DELILAH Experiment on 10 October 1990.

Figure 4. Equivalent Mean Uniform Return Flow at Various Locations on the Barred Profile During DUCK94 Experiment on 11 October 1994.
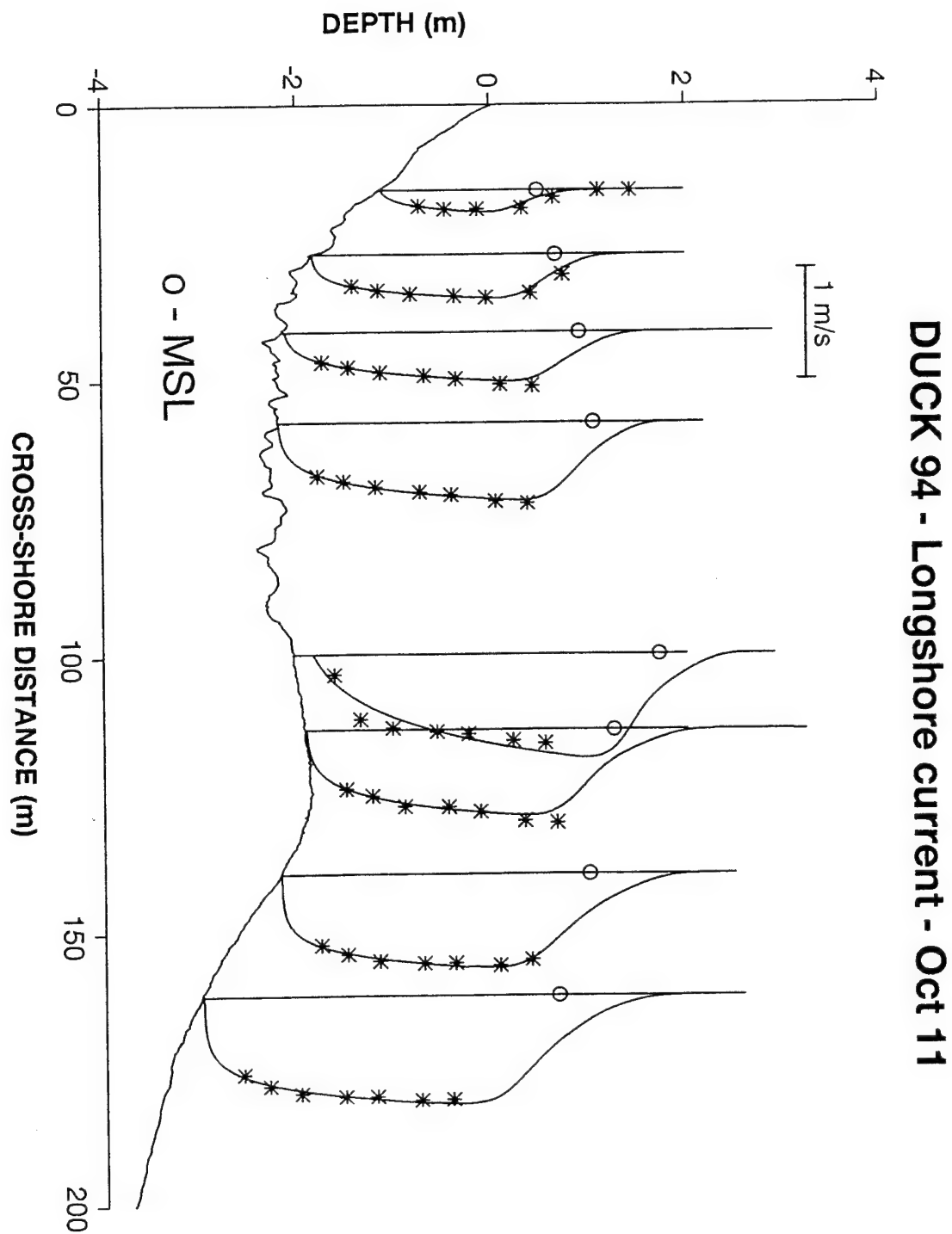
Figure 5. Vertical Profiles of Mean Longshore Current at Various Locations on the Barred

Profile During DUCK94 Experiment on 11 October 1994.

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surface probability distribution function (pdf) is applied to the expected mean current profiles in the absence of waves. The percent of time the current meter is out of the water is given by $1-P(\eta)$, where $P(\eta)$ is the cumulative surface elevation pdf. In an Eulerian frame of reference, the mean current in the crest-trough region is given by

$$V(z) = [1 - P(\eta)]V(z=0) \tag{34}$$

The log profile appears to well describe the measured vertical variation of longshore currents.

A comparison between the predicted longshore currents using the Thornton and Guza (1986) model and the new model that includes the contribution of the momentum flux gradients due to the mean currents is shown in Figure 6 for the DELILAH data on 10 October 1990. A sensitivity test of the model for values of the eddy viscosity coefficient extending over the range of expected values is shown in Figure 7. These preliminary results show that the model is numerically stable and not overly sensitive to changes in the eddy viscosity coefficient.

Comparing predicted longshore currents by this simple three-dimensional model including the momentum flux gradients with the data, it is concluded that the momentum mixing provided by the mean currents, although an important term, is not sufficient to explain the current maximum occurring within the trough of the bar.

This simplified three-dimensional model can be significantly improved by including the following mechanisms:

1. Wave energy and radiation stress gradients based on solving the energy balance with the broken waves described as rollers, using the model of Lippmann and Thornton (1995). The roller results in an additional stress at the surface in the direction of the waves, which will change the

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the wave set-up/down and consequently the forcing terms for both the cross-shore and longshore currents.

2. An eddy viscosity that is allowed to vary both in the vertical and in the cross-shore direction. This variation could be related to predicted roller dissipation. The vertical variation will provide a more realistic undertow model, thus improving the calculation of the momentum mixing by mean currents. The cross-shore variation will take into account the observed variability of the intensity of the onshore mass transport by waves and by the roller of the breaking waves throughout the surf zone.

3. For the undertow modeling, a better formulation for the eddy viscosity variation within the wave (bottom) boundary layer is necessary in order to obtain the more physical no-slip boundary condition at the bottom. The present undertow model over-predicts the currents near the bed, and consequently affects the calculation of the momentum mixing by mean currents. The measurements obtained with a new acoustic instrument, the Coherent Acoustic Sediment Probe (CASP), during DUCK94, can be used to calculate the Reynolds' Stresses. Thus, providing a unique data set to be used to validate a model within the wave boundary layer.

4. A cross-shore variable bottom shear stress due to wave breaking and bottom roughness as suggested by Thornton et al. (1995).

5. Modification of longshore current vertical profile by wind.

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A SIMPLE QUASI-3D MODEL OF LONGSHORE CURRENTS
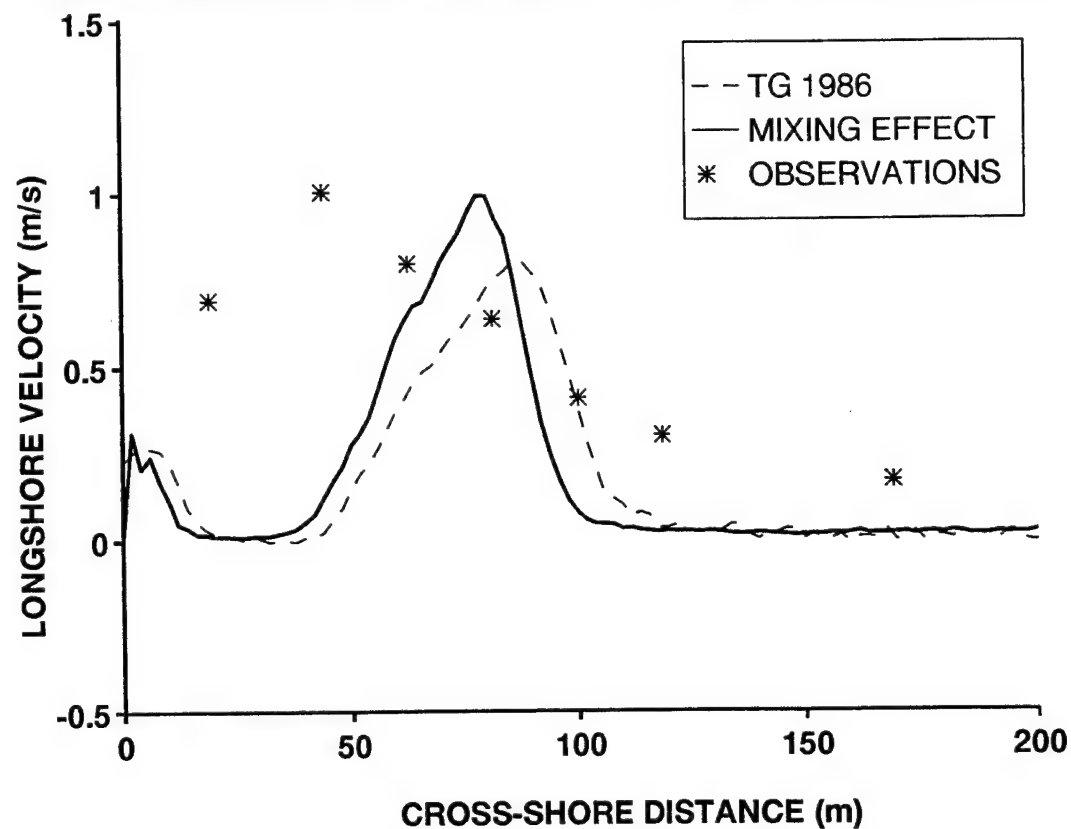

Figure 6. Predicted Longshore Currents Using Thornton & Guza 1986 Model and the New Model that Includes the Contribution of the Momentum Flux Gradients Due to the Mean Currents During DELILAH Experiment on 10 October 1990.

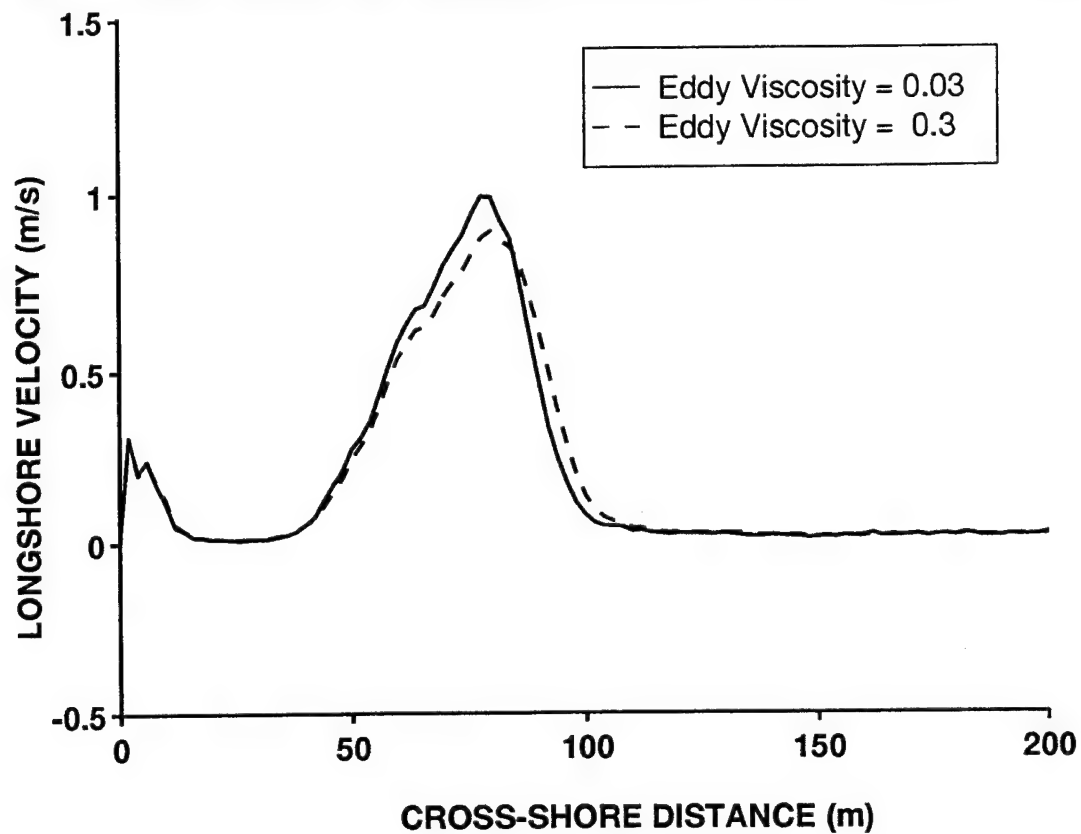# A SIMPLE QUASI-3D MODEL OF LONGSHORE CURRENTS

Figure 7. Predicted Longshore Currents for Two Values of Eddy Viscosity Extending over the Range of Expected Values.

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